Goal Constraints in Insight Problem–Solving

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Abstract

Suzuki & Hiraki (1997) proposed a theory of insight. The theory assumes that three kinds of constraints (object–level, relational, and goal) initially forms impasse, but they are relaxed gradually and independently by the recognition of failure, which probabilistically leads us to insight. According to the theory, the goal constraint is supposed to consists of an image of the goal and gives feedback to problem–solvers. However, previous studies did not provide the evidence for this constraint. In this paper, we empirically examined roles of the goal constraint, using a geometric puzzle (T puzzle). To make the goal constraint operate more explicitly, we gave the template sheet printing the goal image to a half of the subjects. The results showed that subjects with the template sheet solved the puzzle more quickly with fewer errors than those without the template sheet.

Introduction

Research on insight has accumulated experimental evidence on its cognitive processes (Finke, Ward, & Smith, 1992; Smith, Ward, & Finke, 1995; Sternberg & Davidson, 1995). However, there still exists a mystery in insight. Many believe that insight comes to mind by finding "important" cues. But a mysterious thing is that the important cue is often available a lot before insight actually takes place and the neglected cue unexpectedly become illuminative at a certain point.

Kaplan and Simon (1990) gave empirical evidence for this, using the multilated checkerboard (MC) puzzle. It is well known that realizing parity of the differently colored squares is crucial for solving the puzzle. Some subjects in their experiment were given a special board where each square was filled with either "Bread" or "Butter" in checkerboard fashion. Kaplan and Simon hypothesized that these subjects could notice parity more easily and solved the puzzle more quickly, since bread and butter connote parity. Their hypotheses were confirmed. These subjects actually noticed the parity and solved the puzzle more quickly.

However, they reported one puzzling result. The time from the first mention of parity to the final solution were longer for these subjects than those who were given a standard checkerboard or blank one. While subjects given a Bread–Butter board took on average 653 s to solve the puzzle from their first mention of parity, those given a standard checkerboard took 110 s. These results cannot be easily explained by retrieval failure (Ohlsson, 1992), because noticing parity is supposed to activate related operators.

These results show that people fail to use important cues effectively, although they are often found in a relatively early stage of the processes. This indicates the "readiness" of the cue. To put it another, problem–solvers must reach a certain mental state to use important cues effectively.

What leads subjects to be ready to use the cues? Kaplan and Simon (1990) suggested that the parity cue does not work effectively until subjects abandon the covering problem space where they attempt to manually cover the board by dominos. It means that recognition of failure of the initial attempts is the key to use parity information. The importance of failure in insight also is emphasized by Seifert and her colleagues (Seifert, Meyer, Davidson, Patalano, & Yaniv, 1995). According to their opportunistic assimilation hypothesis, when people find that a standard approach does not work, they generate failure indices that mark an initial problem solving attempt and impasse. These failure indices are supposed to have special status in long–term memory, in the sense that they keep activated for longer period than other types of memory traces. In the incubation phase where people stop their initial attempts and are engaged in other activities, relevant cues are sometimes provided externally, which remind their initial failure and lead them to AHA experience.

These studies have revealed that failure plays a key role in readiness for using the important cues. However, the roles of failure must be analyzed more carefully, because it is not always the case that people switch one problem space to another, immediately af-

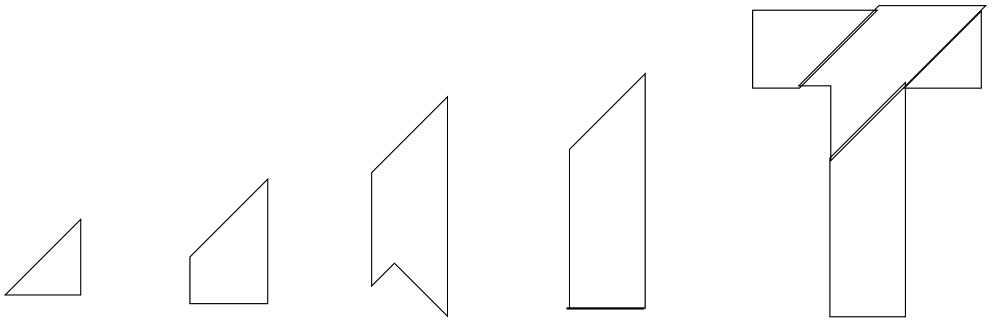

Figure 1 The T puzzle. Construct the shape of "T" using the four pieces on the left hand side.

ter noticing failure. Such a "digital" switch is rather rare in insight problem–solving. As known as (functional) fixation, people usually recognize the failure of a standard approach to an insight problem, but it is awfully difficult to escape from it. Kaplan and Simon reported an episode where a graduate student stuck in a wrong problem space of the MC puzzle, spending 18 hours, leaving a 61 pages long note.

Thus, it is dubious that the failure "triggers" an insight. Rather, the effects of failure seem to be indirect and gradual. It might be that accumulated experiences of failure cause a cognitively unstable state which in turn cause people to be more and more sensitive to useful information previously unnoticed or neglected.

Dynamic Constraint Relaxation in Insight

The above discussion indicates that there is some kind of readiness for using cue, and that failure plays a role in setting the readiness. However, no principled explanation for them has been proposed.

To explore the above issue, we propose a dynamic constraint relaxation theory of insight. The theory consists of three kinds of constraints and a relaxation mechanism. To put it very simply, the theory assumes that the constraints initially forms impasse, but they are relaxed gradually and independently by the recognition of failure. According to the theory, an insight probabilistically takes place as a result of relaxation of the constraints.

The *object–level constraint* determines encoding of objects in the problem. There are more than one ways of encoding, but we have a natural tendency to encode it at the basic level (Rosch, 1978). This greatly contributes to cognitive economy. However, it sometimes leads us to impasses. For example, in the Duncker's "Candle Problem," it is well known that people do not notice a pasteboard box of tacks as a holder of the candle. This is because the basic level of a box is "box," not a "solid body (more abstract)" or a "pasteboard box (more concrete)."

The *relational constraints* correspond to people's natural preferences of how objects in a given problem are related. Like the encoding of objects, there are many ways to relate objects. For example, the pasteboard box in the candle problem can interact with others in the ways of holding something inside, being a platform on which something else stands, being thrown to others, etc. However, there are a few specific relations that connect objects in the problem. The relational constraint, people's preference of relation selection, is affected by the the encoding of the objects and goal. For example, once the pasteboard box in the candle problem is encoded as a "box," the salient relation is not to be a platform, but to hold something inside.

The *goal constraint* evaluates a degree of match of the current state to the goal, and forces problem–solvers to select specific combinations of the representations of objects and relations.

The constraints listed above constrain the way of encoding and relating objects in the problem and constitute an initial problem representation that is natural to most problem–solvers but leads them to the impasse.

The theory assumes that these constraints are gradually relaxed by repeated failures, which increases the frequencies of the constraint violations. For example, to put something inside the box always leads problem–solvers to failure, which, in turn, leads them to encode the box or to relate objects in different ways. An insight probabilistically takes place when a certain set of constraint violations occurs simultaneously.

In our theory, the object–level and relational constraints are implemented as a set of strengths of all the operators. The relaxation algorithm is currently based on a version of the delta rule. However, it differs from a typical algorithm in that relaxation is assumed to occur at multiple levels (object–level, relation, and goal).

This feature is crucial because it partly explains readiness. One of the reason why insight is hard to achieve is that relaxation should operate at multiple levels, and that a specific combination of constraint violation must take place simultaneously. Thus, the relaxation of a single constraint does not always lead problem–solvers to an insight.

In developing our theory, we are greatly influenced by multiconstraint theory of analogy that employs multiple constraint satisfaction as the key mechanism (Holyoak & Thagard, 1989; Thagard, Holyoak, Nelson, & Gochfeld, 1990). In ACME and ARCS, three kinds of internal constraints operate simultaneously to reach a stable state. Unlike these models, our theory is *dynamic* in the sense that constraints are revised not only by other internal constraints, but by the interaction with the external world. We do not claim that every model must be dynamic. But, in many activities, we act to the external world, perceive its change, and take a next action based on the perception. Thus, incorporating interaction is crucial for cognitive models that are concerned with these activities.

Hiraki and Suzuki (1998) and Suzuki and Hiraki (1997) conducted a series of experiments to test the theory. The material was the T puzzle shown in Figure 1. The goal of this puzzle is to construct the shape of the "T" using the four pieces depicted on the left hand side in Figure 1.

At the first glance, it appears to be very easy to solve, since there are only four pieces and the shape to be constructed is quite simple. However, a pilot study, in addition to our own experiences, showed that no one, without having prior experience with this kind of puzzle, could solve it within five minutes. It usually took more than half an hour to solve it. Furthermore, not a few claimed that it was impossible, and some gave up solving it.

In the puzzle, the object–level constraint represents people's tendency to place a piece so that one of its sides (usually the longest one) constitutes a parallel line to the base line (for example, the edge of the desk). The relational constraint are concerned with how one piece is physically connected to others. In the "T" puzzle, the relational constraint corresponds to the tendency to construct a "good form" that has as few angles as possible. These constraints explain why people's placement conform to a relatively small number of patterns among the infinite number of combinations.

According to the previous studies (Hiraki & Suzuki, 1998; Suzuki & Hiraki, 1997), most subjects attempted to separately construct the horizontal and vertical bar of "T." This leads them to use the pentagon as a part of either horizontal or vertical bar, which causes them to place it horizontally or vertically. Once the pentagon is placed as such, it is necessary to fill the notch (the swallow–tailed part) of the pentagon so as to form the straight bar. Thus, the impasse in solving the puzzle was attributed mainly to the object–level constraint (placing the pentagon piece horizontally or vertically) and the relational constraint (filling the notch of the pentagon with other pieces). In more than 70% of trials, subjects tried to place the pentagon horizontally or vertically and to fill its notch. In order to find evidence for constraint relaxation, we divided an entire problem–solving process of each subject into the first and second halves and compared the frequencies of constraint–violating placements of the pentagon. The results showed that the number of the constraint violation in the first half was twice as many as that in the second half. This strongly supports our claim that the object–level and relational constraints be relaxed in the course of problem–solving.

Goal Constraint

The goal constraint is crucial because it gives subject the image of the desired state and feedback by the match between current and goal states. However, we obtained ambiguous results about the role of the goal constraint. In one of the experiments (Suzuki & Hiraki, 1997), subjects were given a sheet of a paper printing "T" in the same size as the actually constructed "T." They were instructed to cover the printed "T" using the four pieces. It was expected that introducing such a template gave subjects a clear image of the goal and effective feedbacks, which led them to insight. However, this manipulation facilitated some subjects' performance but not others. Half of the subjects could not solve the puzzle without a hint.

There are several reasons for it. First of all, the puzzle is still difficult even when the template is available. Since the three types of constraints should operate in a coordinated fashion, emphasizing a single constraint may not be sufficient. Second, the number of the subjects was too small to detect the effects of the goal constraints. Since individual differences are so apparent in this kind of problem–solving, the subtle effects may not be found if the number of subjects is small.

Experiment

An experiment was conducted to examine the role of the goal constraint. In order to make the puzzle easier, we fixed one of the pieces at the proper position on the template sheet. If subjects who were given this modified puzzle outperform those who solved the same puzzle without a template sheet, we can attribute the differences to the goal constraint.

Table 1 Solution time

	MIN	MAX	Median
Template			
big trapezoid fixed (FbT)	10	962	145.5
small trapezoid fixed (FsT)	15	410	79
triangle fixed (FtT)	14	702	105
No–Template			
big trapezoid fixed (Fb)	754	1269	1096.5
small trapezoid fixed (Fs)	226	1242	952.5
triangle fixed (Ft)	460	1271	1098

Table 2 The number of segments

	MIN	MAX	Median
Template			
big trapezoid fixed (FbT)	1	98	16.5
small trapezoid fixed (FsT)	1	43	3.5
triangle fixed (FtT)	1	60	7
No–Template			
big trapezoid fixed (Fb)	34	118	68.5
small trapezoid fixed (Fs)	13	133	52
triangle fixed (Ft)	55	76	67

Method

Subjects: Subjects were 40 university students. Some of them had seen the puzzle, but none of them had any prior success on this puzzle. These subjects were randomly assigned to one of the six conditions described below.

Design: The experiment had 2 (the template sheet given or not) $\times$ 3 (kinds of pieces fixed, the big and small trapezoid, and triangle) between–subjects design. Subjects were assinged to one of the six conditions, fixed-big-trapezoid with the template sheet (FbT), fixed-small-trapezoid with the template sheet (FsT), fixed-triangle with the template sheet (FtT), fixed-big-trapezoid (Fb), fixed-small-trapezoid (Fs), and fixed-triangle (Ft).

We did not include a condition where the pentagon was fixed. It was because a previous study showed that, even without the template sheet, the effects of fixing the pentagon was so powerful that every subject could complete the puzzle within a minute (Suzuki & Hiraki, 1997).

Procedure In the template conditions (FbT, FsT, and FtT), subjects were given a template sheet with one of the three pieces fixed at the proper position and asked to cover the shape of the "T" using remaining three pieces.

In the non–template condition (Fb, Fs, and Ft), one of the three pieces were fixed at the proper position. Subjects were asked to construct the shape of "T" using the remaining three pieces. They were told not to move the fixed pieces because it had been already placed at the proper position and in the proper orientation.

If subjects could not solve the puzzle within 15 minutes, the experimenter told the subjects not to fill the notch of the pentagon.

We did not force subjects to think aloud, because verbalization sometimes interferes insight (Schooler, Ohlsson, & Brooks, 1993). The entire performance was videotaped in order for the later analysis.

Results

We analyzed subjects' performance with respect to time to solve and the number of segments.

Solution time

Solution time varied from 10 seconds to more than 20 minutes. Table 1 shows the minimum, maximum, and median solution time in each condition.

As obvious in Table 1, providing a template sheet greatly reduced the solution time. The distributions of corresponding conditions (FbT vs. Fb, FsT vs. Fs, and FtT vs. Ft) have little overlap.

To show the effect in more detail, we counted the number of subjects completed the task by every five minutes. Table 2 shows the proportion of subjects who solved the task within 5, 10, 15 minutes and those who were given the hint. More than 70% of subjects in the template conditions solved the puzzle within five minutes, while about three quarters of subjects in the no-template conditions took more than 15 minutes to solve. Furthermore, a half of the subjects in the no-template conditions were given the hints, while only 5% subjects in the template conditions received the hint. These results clearly show

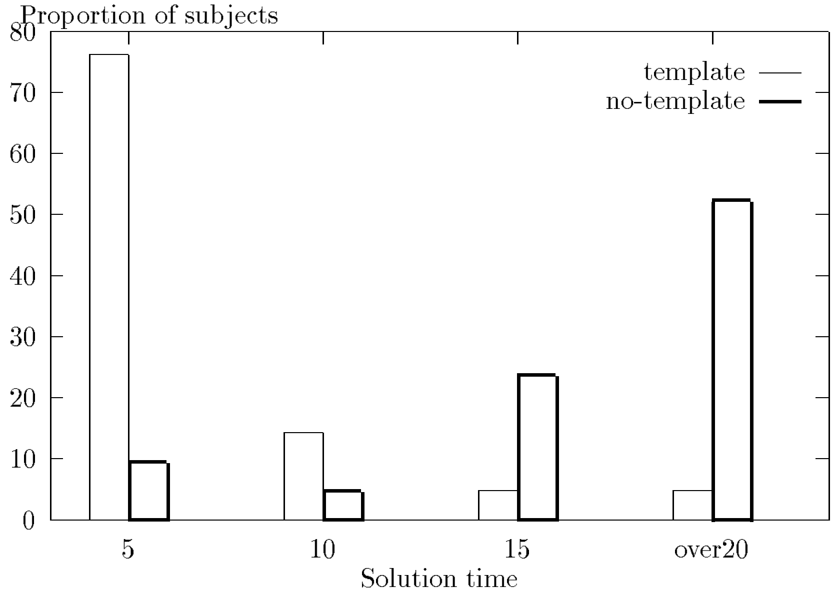

Figure 2 Distributions of the solution time.

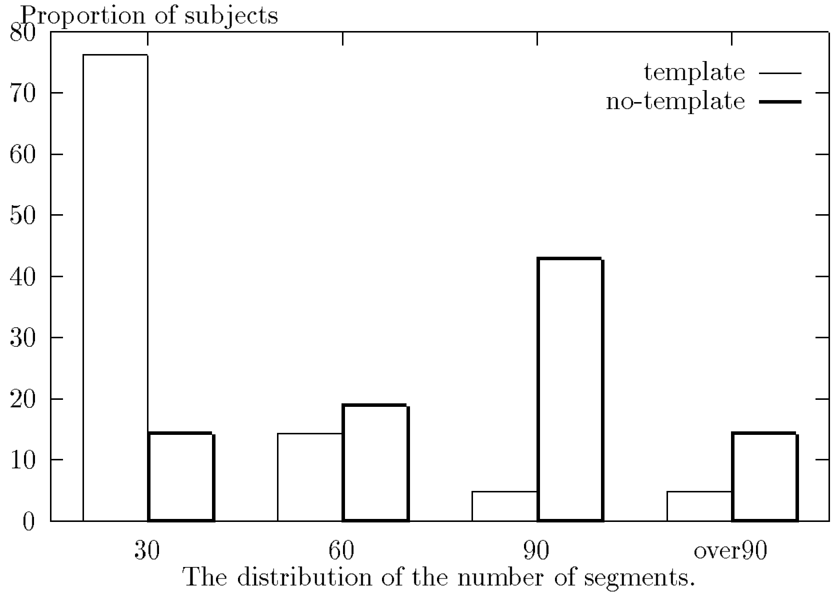

Figure 3 The distribution of the number of the segment.

that providing the template sheet has profound effects on solving the puzzle.

The number of segments

We sliced an entire problem–solving process of a subject to segments. A segment roughly corresponds to one trial that begins with connecting pieces and ends up with noticing failure or achieving the goal. A segment is operationally defined as a series of actions that was initiated by joining two pieces and terminated by their separation. Thus, an action such as adding another piece to joint pieces does not constitute a segment, but is regarded as an action within a segment.

Table 2 shows the minimum, maximum, and median numbers of segments in each condition. As in the solution time, the subjects in the template conditions needed fewer trials to reach insight than those in the no–template conditions.

To analyze the data in more detail, we counted the number of subjects who completed the task by every 30 segments. Table shows the distribution of each condition. The number in the second and third row shows the percentages of the subjects who could solve the task within the given number of segments shown in the top row. The distributions of the three template conditions have little overlap with those of the no–template conditions. More than 90% subjects in the template conditions completed the task within 60 segments, while only one third of the subjects in the no–template conditions did so. Furthermore, about a half of the subjects in the template conditions required less than five segments. This means that these subjects made few errors.

General Discussion

We propose the dynamic constraint satisfaction theory of insight. The theory consists of three constraints, object–level, relational, and goal, and assumes that insight takes place by the relaxation of these constraints.

In the experiment, we empirically examined the roles of goal constraints in insight problem–solving. Subjects in the template conditions were given the template sheet that represents the clear goal image, while those in the no–template conditions were not. We predicted that the former subjects outperformed the latter, because the template facilitates matching between current states and the goal and gives feedback more effectively.

The results showed that subjects performance dramatically improved by giving the template sheet. Those who were provided with the template solved the puzzle more quickly with fewer errors than those without the template.

These differences are partly due to the ease of matching. A non–negligible number of the constraint–violating placement of the pentagon were observed in the no–template conditions even in relatively early period of their problem–solving. However, without the template, it is hard for subjects to recognize which part of the "T" the pentagon placed as such occupies. On the other hand, subjects with the template sheet could match the pentagon placed in a constraint–violating manner to the shape of "T" and realize that the notch needs not be filled.

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